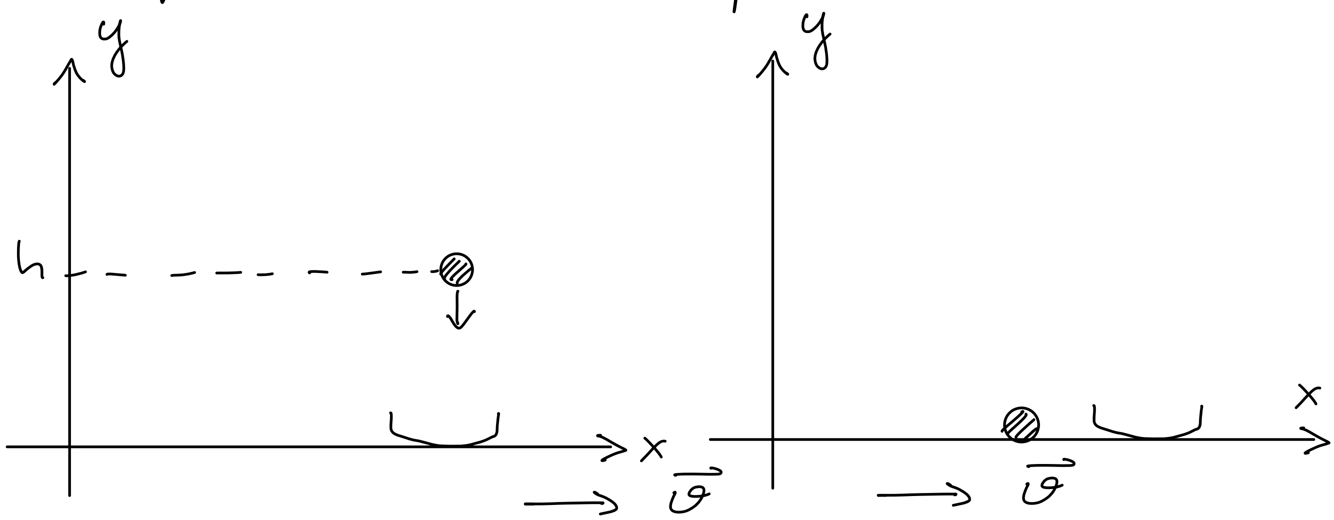
§3. Newton's Laws

(Isaac Newton ~ 1642 – 1726)

§3.1 Inertial Frames

In acient times philosophers argued that the earth cannot move around the sun as in that case falling objects would be displaced:

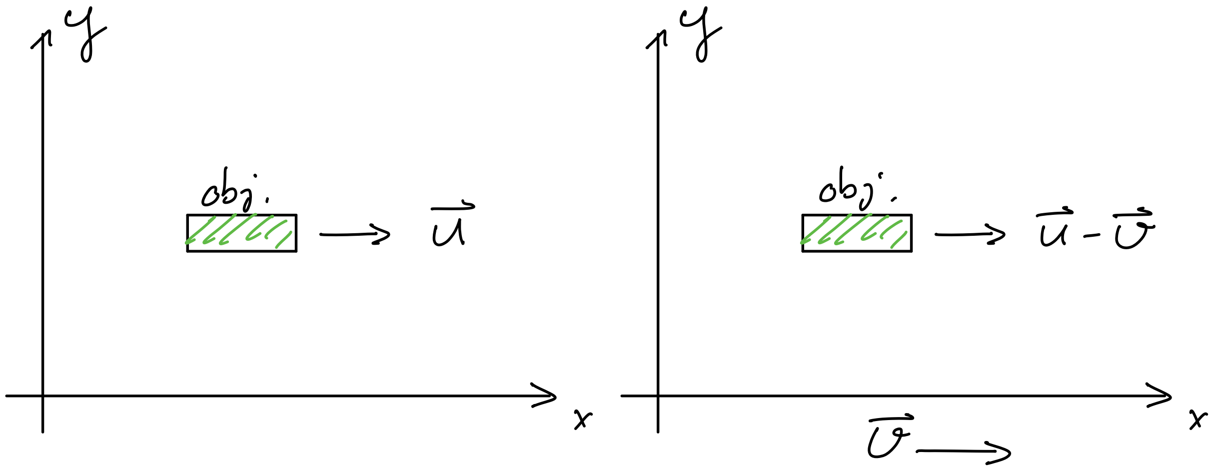


Galileo was the first to realize that physical laws are invariant in "inertial frames" in uniform motion with respect to each other

Definition 1 (inertial frame):

Intertial frames are coordinate frames

which move with constant velocity relative to each other and in which <u>Newton's laws</u> are valid.



Laws of physics are the same for the object in the above two frames (only its velocity is different)

<u>Newton's 1st law</u> (law of inertia):
"If a body has no "forces" acting on it, it will maintain its velocity."

The earth is an approximate inertial frame:

- acceleration due to motion around sun: $a = v^2/r = 0.006\ ms^{-2}$
- acceleration due to spin: $0.03\ ms^{-2}$

Newton's second law:

"If a body has an acceleration $\vec{a}$, then you need a force

$$\vec{F} = m\,\vec{a},$$

to produce that acceleration."

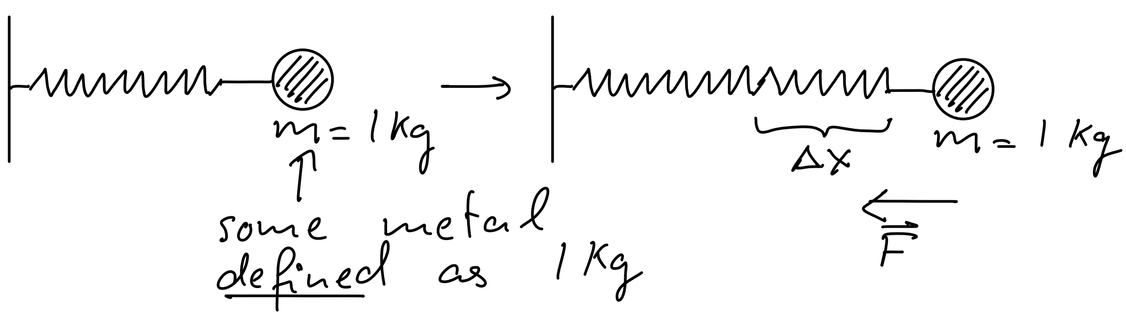
Remarks:

1) "Mass" m is measured in Kilograms kg
2) Acceleration is measured in ms^{-2}
3) Force $\vec{F}$ is measured in $kg\, m\, s^{-2} =:$ Newton or N

Question:

How do we measure mass and force?

Imagine a spring attached to a wall:



If we elongate the spring, it will exert a force $\vec{F}$ on the mass m

$\longrightarrow$ measure initial acceleration a_1 after release

If we now attach another mass M to the elongated spring, then we get:

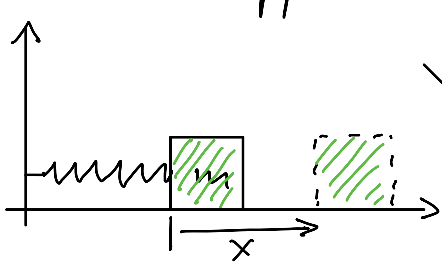
$$1\,\text{kg} \cdot a_1 = M\, a_M$$

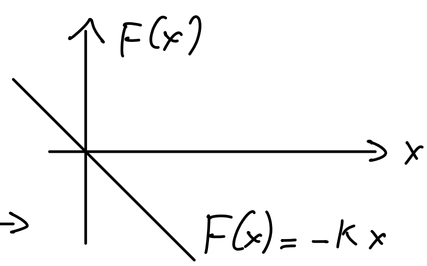
$$\Leftrightarrow \quad M = 1\,\text{kg} \cdot \frac{a_1}{a_M}$$

$\longrightarrow$ to measure M, we just have to measure a_M!

<u>Example 1</u> (application of 2nd law):

If a spring is stretched by a distance x, then it exercises a force $F = -kx$ in the opposite direction





$\rightarrow$ inserting into $F = ma$ gives: $m\frac{d^2x}{dt^2} = -kx$

$\longrightarrow$ solving gives $x(t)$

Newton's third law:

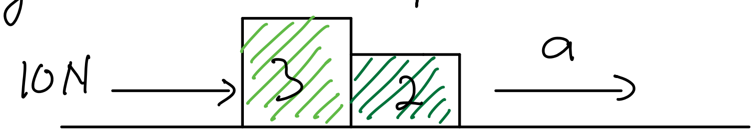
If there are two bodies, called 1 and 2, then F_{12}, the force on 1 due to 2, is minus the force on 2 due to 1:

$$F_{12} = -F_{21}$$

"action = reaction"

Example 2 (application of 3rd law):

Imagine a 3-kg block packed against a 2-kg block, and push the 3-kg block with 10N:

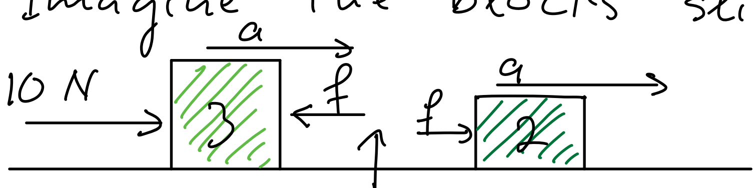


Want to determine a

i) total mass = 3kg + 2kg = 5 kg

$$\Rightarrow \quad a = \frac{F}{m} = \frac{10\,N}{5\,kg} = 2\,ms^{-2}$$

ii) Imagine the blocks slightly displaced:



follows from 3rd law!

Plugging into $F = ma$ gives:

$$\left.\begin{aligned} 10N - f &= 3kg \cdot a \\ f &= 2kg \cdot a \end{aligned}\right\} \begin{array}{l} (*) \\ \text{both move with} \\ \underline{\text{same}} \text{ acceleration} \end{array}$$

adding the above equations gives:

$$10\,N = 5\,kg \cdot a$$
$$\Leftrightarrow \quad a = 2\,ms^{-2}$$

$\longrightarrow$ inserting back into $(*)$ gives:

$$f = 4\,N, \qquad (10-4)N = 6\,N$$

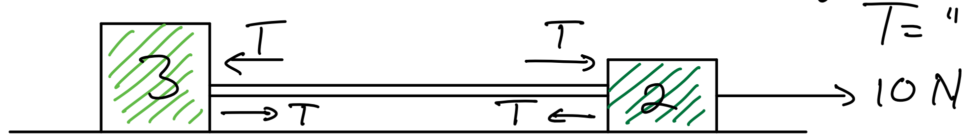
$\downarrow$ acting on 2kg mass

$\downarrow$ acting on 3kg mass

<u>Example 3</u> (another application):

Imagine the same 3kg and 2kg blocks, but now connected by a rop:

T= "tension"

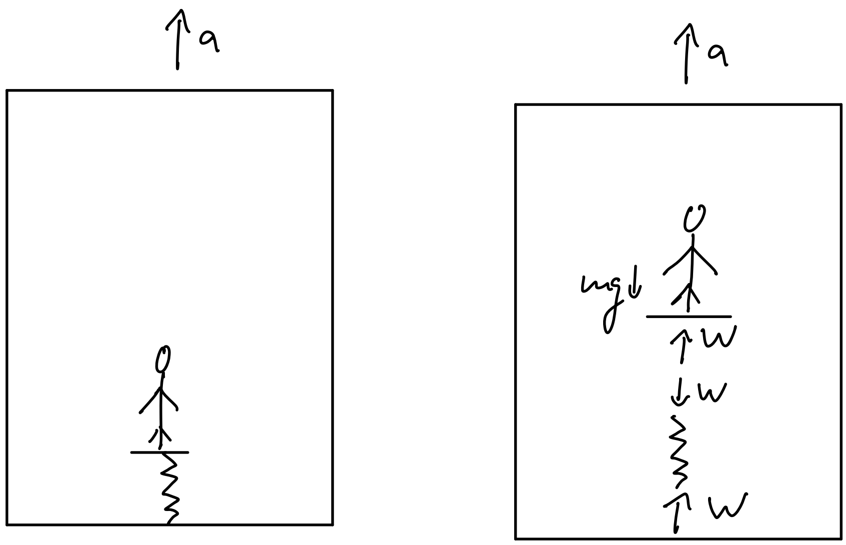


mass 2 is being pulled with 10 N

$$\left.\begin{array}{l} \Rightarrow T = 3a, \\ T - T = \underset{\uparrow}{0} \cdot a \\ \quad \text{rope nearly massless} \\ 10 - T = 2a \end{array}\right\} \begin{array}{l} \text{adding gives} \\ 10 = 5a \quad \text{or} \\ a = 2\,\mathrm{m\,s^{-2}} \\ \Rightarrow T = 6\,N \end{array}$$

Example 4 (weight and weightlessness):

Imagine you are standing on a spring in an elevator which is accelerating upwards:



Every (massless) spring is pushed or pulled by equal and opposite forces $\pm F$ at the two ends since otherwise it would have $a = F/0 = \infty$.

→ relevant equations:

$$W - mg = ma$$

$$\Leftrightarrow \quad W = m(g+a)$$

- if the elevator is standing still, then $a = 0 \Rightarrow W = mg$
  "you feel your own weight"
- if accelerating upward, then
  $a > 0 \Rightarrow W = m(g+a)$
  "you feel heavier"
- if accelerating downward, then
  $a < 0 \Rightarrow W = m(g - |a|)$
  $< mg$
  "you feel lighter"
- if the elevator is freely falling towards the ground, then
  $|a| = g \Rightarrow W = 0$
  "you feel weightless"

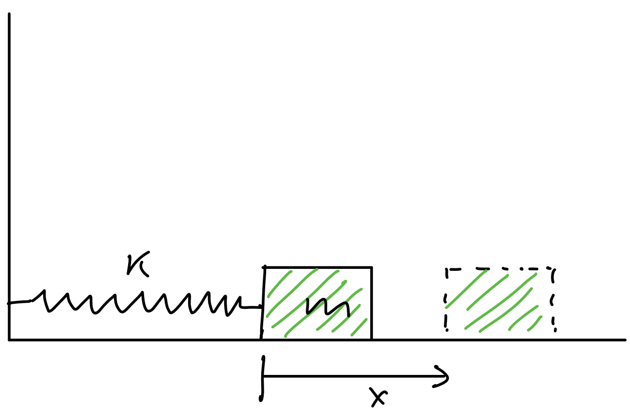
<u>Note</u>: you have <u>not</u> escaped the pull of gravity !

§ 3.2 Solutions to Newton's equations

Goal of physics:
predict the future based on present information about location, velocity, mass and forces acting on objects

→ In this paragraph we want to demonstrate an example of this paradigm:

Imagine a frictionless table with a mass m attached to a spring:



→ combining $F = ma$ and $F = -kx$, we get

(*) $$\frac{d^2x}{dt^2} = -\frac{k}{m}\,x(t)$$ "differential equation"

→ mathematics tells us that this equation has a unique solution given initial information

- $x(0)$: initial position x_0
- $\left.\frac{dx}{dt}\right|_{t=0}$: initial velocity v_0

→ Let's find it!

We find that $\cos\omega t$ and $\sin\omega t$ with $\omega = \sqrt{\frac{K}{m}}$ are two linearly independent solutions of (*)

→ make general ansatz:

$$x(t) = A\cos\omega t + B\sin\omega t$$

evaluate;

- $x(0) = A = x_0$
- $\left.\frac{dx}{dt}\right|_{t=0} = \left. -A\omega\sin\omega t + B\omega\cos\omega t\right|_{t=0}$

$$= B\omega \overset{!}{=} v_0$$

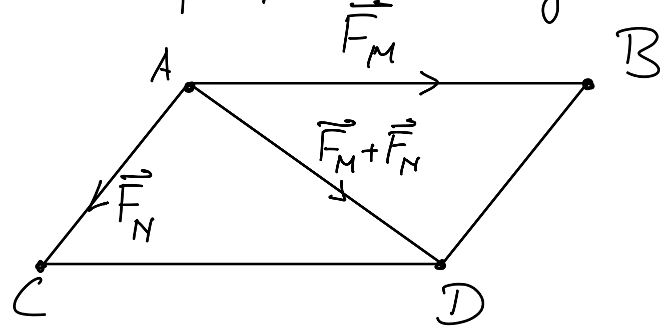
$$\Leftrightarrow B = \frac{v_0}{\omega}$$

In our case: $v_0 = 0 \Rightarrow B = 0 \Rightarrow x(t) = x_0\cos\omega t$

§ 3.3 Motion in $d = 2$

Corollary 1 (to Newton's 2nd law):

A body pulled by two forces $\vec{F}_M$ and $\vec{F}_N$ along different angles will move along the diagonal of a parallelogram as if pulled by a force $\vec{F}_M + \vec{F}_N$:



$$AB = \frac{1}{2}\frac{\vec{F}_M}{m}T^2$$

$$AC = \frac{1}{2}\frac{\vec{F}_N}{m}T^2$$

Proof:

Suppose the object would move in time T by force $\vec{F}_M$ from A to B, and by force $\vec{F}_N$ from A to C. Then the simultaneous application of force $\vec{F}_N$ will not alter the velocity along AB but only in the direction of $\vec{F}_N$

$\rightarrow$ object will arrive somewhere on BD

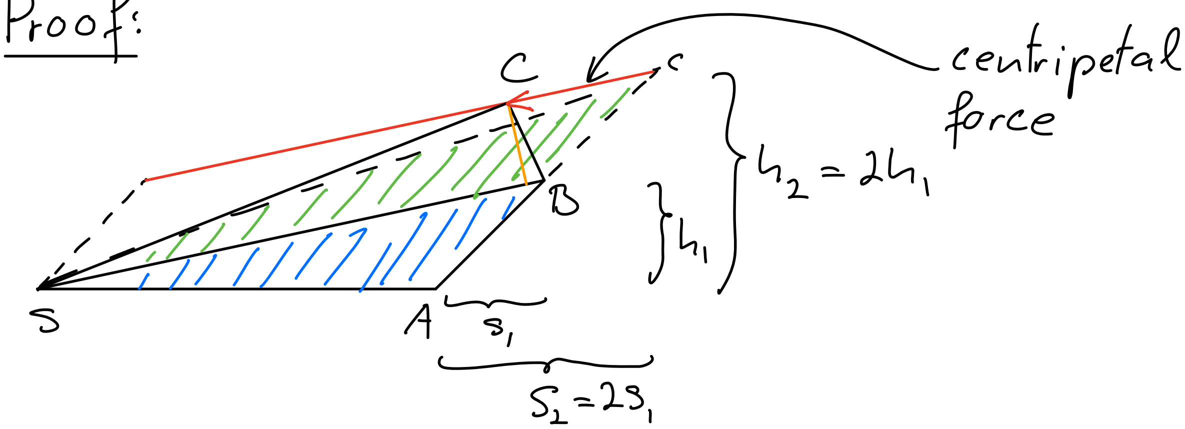
similarly $\rightarrow$ will arrive somewhere on CD

$\rightarrow$ arrives at D $\square$

Theorem 1 (Area law):

Suppose a massive point-like object is subject to a central force. Then its motion is contained in a plane and the radius vector sweeps through equal areas in equal times

Proof:



$$\text{area}(SAc) = \frac{1}{2}(SA + 2s_1)\cdot 2h_1 - \frac{1}{2}\,2s_1\,2h_1$$

$$= \frac{1}{2}\,SA\,2h_1 + \cancel{s_1 2h_1} - \cancel{2 s_1 h_1}$$

$$= SA\,h_1$$

$$\text{area}(SAB) = \frac{1}{2}(SA + s_1)\cdot h_1 - \frac{1}{2}\,s_1 h_1$$

$$= \frac{1}{2}\,SAh_1 + \cancel{\tfrac{1}{2}s_1 h_1} - \cancel{\tfrac{1}{2}s_1 h_1} = \frac{1}{2}\,SAh_1$$

$$\Rightarrow \text{area}(SBc) = \frac{1}{2}\,SAh_1$$

Use $\text{area}(SBC) = \text{area}(SBc) \rightarrow$ claim follows $\square$